A post-reconstruction comparison of denoising methods for micro-CT data

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Aims
A well-known problem of micro-computed tomography (µCT) systems is the unwanted noise in the reconstructed images for small pixel sizes (i.e. micrometre range). In this study we compare conventional methods, including Gaussian smoothing and median filter, with novel total variation algorithms for image denoising. The reconstructions were performed using the modified Feldkamp algorithm and subsequently denoised with different algorithms. The methods were evaluated first on a simulated 3D Shepp-Logan phantom and subsequently applied to a physical phantom. In order to evaluate the efficacy of the denoising techniques, a quantitative analysis was performed on the reconstructed images. Signal-to-noise ratio was up to 12-times higher and to contrast-to-noise ratio increased up to 10-fold using a denoising algorithm based on $L_2$ norm for isotropic total variation compared to the conventional Feldkamp reconstruction.

I. Introduction
X-ray µCT has emerged as a powerful tool due to study (non-) biological samples to its non-destructive nature and the ability to investigate specimens in three dimensions (3D) [1]. The performance and the quality of a cone-beam µCT (CBµCT) system are strongly influenced by multiple factors, such as: photon scatter, scanner geometry, scanner misalignment, X-ray spectrum, detector pixel size, voxel size, number of projections, reconstruction algorithm, electronic noise and quantum noise [2-4]. While the majority of the undesired effects caused by these factors can effectively be corrected, random noise remains an unresolved issue impairing the image quality. Thus, the measurement of various parameters, such as volumes or texture, which is performed directly on the reconstructed images, can be compromised if a significant noise level is present. Furthermore, segmentation and more advanced image analysis, such as the automated extraction of (micro-) structures, can be hampered, necessitating the use of effective denoising algorithms as a pre-processing step.

Two different denoising schemes are possible in principle in order to obtain reliable tomograms: pre-reconstruction denoising applied directly on the sinograms in order to explore statistical properties of the noise available in the sinogram-domain, or in order to explore the image characteristics rather than the data properties a post-reconstruction denoising step is preferred [5]. In commercial µCT systems, computationally but simpler denoising methods are implemented, such as linear smoothing filters (symmetrical, asymmetrical or Gaussian [6]). Since the power of the noise is concentrated in high and medium frequencies [7], these types of filters cause blurring of fine structures, affecting the sharpness of edges. In order to preserve edges, more complex techniques, such as wavelet techniques with soft thresholding, shrinkage schemes, adaptive or non-linear filters (e.g. the medium filter which perform well for impulsive noise) can be applied. In this study, we investigate two approaches, which are based on total variation (TV) image denoising models proposed in [8], for the hitherto unreported application to µCT data. By using these models a denoised image can be seen as
the proximity operator of the TV calculated from a given noisy image. The convergence of the algorithms is achieved by using a fixed-point methodology specialized for these TV models. While these TV models were proposed for removing additive white Gaussian noise, the performance of these algorithms for denoising µCT-data (with non-Gaussian noise distributions) is not known.

II. Method
A. Total variation µCT image denoising
Under the assumption that a µCT tomogram has a piecewise constant structure a TV model known also as Rudin-Osher-Fatemi (ROF) [9] can be applied. TV was reported in the literature as a noise reduction and deblurring method [10-11] and a large number of variants of this model have been proposed. The merit of this regularization model is that the edges are preserved in the denoised images. The constrained minimization problem considered in the ROF model is given by:

$$ f(u) = \arg \min \left\{ \frac{1}{2} \| u - z \|_2^2 + \alpha \| u \|_{TV} : u \in \mathbb{R}^d \right\} \quad (1) $$

where $u \in \mathbb{R}^d$ is the desired true solution, $z$ is the observed or noisy data, $\alpha > 0$ is the regularization parameter and $\| u \|_{TV} = \| \nabla u \|_1$ is the total variation of $u$. Due to the non-differentiability of the total variation norm and the large dimension of the underlying images, the functional from Eq. 1 is difficult to minimize by conventional methods. In order to address this issue, a variety of optimization algorithms have been proposed (see [8] and the reference cited therein). A recent alternative to the ROF model to rewrite Eq.1 as:

$$ f(u) = \arg \min \left\{ \frac{1}{2} \| u - z \|_2^2 + (\varphi \circ B)(u) : u \in \mathbb{R}^d \right\} \quad (2) $$

where $\varphi$ is a convex function on $\mathbb{R}^m$ and $B$ is a $m \times d$ matrix defined as:

$$ B := \begin{bmatrix} I_N \otimes D \\ D \otimes I_N \end{bmatrix} \quad (3) $$

where $\otimes$ denotes the Kronecker product, $I_N$ the $N \times N$ identity matrix and $D$ the $N \times N$ matrix:

$$ D := \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ \vdots & \ddots & \ddots \\ 0 & \cdots & -1 & 1 \end{bmatrix} \quad (4) $$

As it has been shown in Sec. 4 in [8] by choosing $\varphi : \mathbb{R}^{2N^2} \rightarrow \mathbb{R}$ the isotropic total variation (ITV) can be defined as a linear combination of the norm $\| \cdot \|_2$:

$$ \varphi(x) := \alpha \sum_{i=1}^{N^2} \left\| \frac{x_i}{x_i N^2 + i} \right\|_2, \quad x \in \mathbb{R}^{2N^2}, \quad (5) $$

while the anisotropic total variation (ATV) is defined as the composition of the norm $\| \cdot \|_1$:

$$ \varphi(x) := \alpha \left\| x \right\|_1, \quad x \in \mathbb{R}^{2N^2}. \quad (6) $$

By viewing the TV-norm as a composition map, the explicit form of the proximity map of the $l_1$ or $l_2$ -norm can be fully exploited.

B. Simulations
A 3D Shepp Logan phantom consisting of a series of ellipsoids was first simulated in order to evaluate the denoising approaches. 600 projections over 180 degrees were calculated in a cone-beam geometry at 60 kVp. The nominal resolution used for this simulation was 1 µm.
After applying the FDK algorithm, 512×512 pixels cross-sections were obtained. The central slice of the phantom shown in Fig.1(a) was used for comparison as the “ground-truth” reconstruction. In addition, data sets with different levels of Poisson and Gaussian noise were generated. For each noise level, the regularization parameter α was determined empirically and fixed.

C. Experimental data
All µCT-data were obtained on Bruker SkyScan 1172 (Bruker, Kontich, Belgium). A plastic phantom of diameter 36 mm containing a various internal structures surrounded by copper sulphate solution [CONC] was scanned over 180° with 0.15 degree rotation step at 80 kVp (source current 124 µA) at a nominal resolution (pixel size) of 2 µm, using an Al+Cu filter. The frame averaging parameter was set to 6 and the random movement parameter to 10. While setting these two parameters to large values resulted in long acquisition times (i.e. 14 hours), this compromise was made in order to acquire the best image quality. The projection size was 2672 x 7744 using the 1x1 camera binning. The reconstruction parameters were: beam hardening correction 43% and ring artifact 20%. The cross-section size was 7744×7744 and 411 slices were reconstructed in 47 minutes using the reconstruction algorithm provided by the manufacturer NRecon software.

D. Assessment of reconstructed image accuracy
The performances of the TV denoising methods were first evaluated on a 3D Shepp Logan phantom. Since the ideal reconstruction were available (Fig.1(a)) quantitative measurements of the errors were made using normalized mean square error (NMSE) with the $l_2$ norm:

$$\text{NMSE} = 100 \times \frac{||i - r||_2}{||i||_2}$$

(7)

where $i$ is the ideal reconstruction image and $r$ the denoised reconstructed image obtained using a denoising algorithm. The quality of the denoised tomograms were evaluated also using the peak-signal-to-noise ratio (PSNR) defined as:

$$\text{PSNR} = 20 \log_{10} \left( \frac{255}{||i - r||_2} \right).$$

(8)

For the experimental data, the overall quality of the reconstructed images was quantitatively assessed in terms of their signal-to-noise (SNR) and contrast-to-noise ratio (CNR), defined on the tomographic slices of the object as:

$$\text{SNR} = \frac{|I|}{\sigma},$$

(9)

and

$$\text{CNR} = \frac{|I_A - I_B|}{\sqrt{\sigma_A^2 + \sigma_B^2}},$$

(10)

where $I_A$ and $I_B$ denote the mean pixel values, $\sigma_A$ and $\sigma_B$ the standard variations within two selected homogeneous regions respectively. The iterations in the TV denoising algorithms were terminated when the following condition was satisfied:

$$\frac{|r_n - r_{n-1}|_2}{|r_n|_2} \leq \text{TOL}$$

(11)

where $r_n$ is the denoised image at iteration $n$.

III. Results
A. Simulated data
Representative central slices from the simulated Shepp Logan phantom are shown in Fig.1. The noisy images with Poisson noise (PSNR=13.43 dB) used as initialization for the denoising algorithms is displayed in Fig.1(b). The resulting image, denoised with the ITV approach ($\alpha = \ldots$)
35.8) is shown in Fig.1(c), and is comparable to the “ground truth” image (Fig.1(a)). This is confirmed in the profile plots (Fig.1(d)), depicting cross-sections through a homogeneous region of the phantom. It can be observed that the borders of the analysed region are well preserved, and only small deviations from the ideal reconstruction are present. Very similar results were obtained when Gaussian noise was added (data not shown). In this case the simulation time was approximatively the same as for Poisson noise, but the value of the regularization parameter α was smaller. The best results were obtained using the ITV algorithm as summarised in Table II. The PSNR of the initial Shepp Logan slice was 10.2 dB and after 15.6 seconds a denoised image with PSNR=31.7 dB and NMSE =2.6% was obtained. The ITV and ATV approaches reduced the global error of the reconstructed images compared to the initialization solutions. The values of the normalized mean square errors and the peak-signal-to-noise ratios (Table I and Table II) were significantly improved by both denoising algorithms. From the numerical results it can be observed that the ITV and the ATV models gave similar results, but that the ITV method converged faster. Thus, this algorithm was subsequently applied to the experimental µCT data.

![Fig. 1. Tomographic central slice in a Shepp Logan phantom: (a) noise-free image, (b) Image with Poisson noise added (PSNR=13.4). (c) Noisy image reconstructed with ITV method](image-url)
(PSNR=29.6 and NMSE=3.3%) and (d) profile lines over a homogeneous region of the phantom. The profile of the ideal reconstruction of the central slices is shown in red, the noisy profile in black and the denoised profile of the region of interest in blue, respectively.

**TABLE I:** Normalized mean square error (NMSE), peak-signal-to-noise ratio (PSNR) and execution time, obtained after applying ITV and ATV algorithms for Shepp Logan phantom with Poisson noise (PSNR=13.4 dB).

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**TABLE II:** Normalized mean square error (NMSE), peak-signal-to-noise ratio (PSNR) and execution time, obtained after applying ITV and ATV algorithms for the Shepp Logan phantom with Gaussian noise (PSNR=10.2 dB).

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**B. Experimental results**

The original FDK reconstruction of the central slice with the acquisition and reconstruction parameters described in Sec.II-C is displayed in Fig.2(a). The corresponding slice denoised using the ITV algorithm (for α=100 and TOL=0.002) is shown in Fig.2(b). Two homogeneous regions of the phantom were chosen in order to evaluate the performance of the algorithm. These two zones are displayed in Fig.2(b) as a blue square (region A) and a red circle (region B). The standard deviation (SD), SNR and CNR are reported in Table III. Compared to the modified FDK filtered image, the application of the ITV algorithm increased the SNR in region A by a factor of 6.8 and by a factor of 12.3 in region B, respectively. Thus, this resulted in a 8.2-fold higher CNR.
Table III: Numerical results (standard deviation (SD), signal-to-noise ratio (SNR) and contrast-to-noise ratio (CNR)) obtained for the phantom.

<table>
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<tr>
<td>B</td>
<td>FDK reconstruction</td>
<td>32.5</td>
<td>1.6</td>
<td></td>
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<tr>
<td></td>
<td>ITV</td>
<td>2.6</td>
<td>19.8</td>
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Conclusion

In this work, we compared different post-reconstruction denoising techniques suitable for commercial µCT data. The methods were evaluated quantitatively on simulated and experimental data acquired using a commercial µCT system (SkyScan 1172). The proposed algorithm based on an isotropic total variation model significantly improved the Feldkamp reconstructions, and achieved better results in terms of signal-to-noise ratio and contrast-to-noise ratio for both simulated and experimental data. The ITV method decreases globally the reconstruction errors compared to the Feldkamp reconstructions including the smoothing option available as standard in the commercial software of the µCT system. Applied directly on the Feldkamp solutions, the proposed denoising algorithm improved the uniformity of the reconstructed tomographic images, and, importantly, facilitated an automated and accurate extraction of the 3D vascular network of an ex vivo rat heart using a novel 3D vascular reconstruction algorithm. This 3D network reconstruction was not possible using any of the other µCT data sets, reconstructed with FDK, Gaussian smoothed or median filtered, due to the high level of noise.

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References:
[6] NRecon user guide